Impasse, Conflict and Learning of CS Notions

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32 slides

Learning

Rote Learning

Learning with understanding

Procedural knowledge

Conceptual knowledge

Two Examples

Average:

→ How to compute it

→ What does it mean

Iterative Computation:

→ How is it executed

→ What are its characteristics

Average

- → Compute the avg of N nums
- → Given N-1 nums and avg find the N-th num
- → Given K nums and avg offer N-K additional nums
- → Characterize the avg in terms of the nums larger and smaller than it

Iterative Computation

→ Construct a loop to compute ...

→ Given the following loop, offer:

- input(s) that yields no iterations
- input that yields K iterations
- input that yields infinite iterations
- a general relationship (e.g. invariant) between its variables

Notion Utilization

Different types of tasks:

Explicit reference to the notion

No explicit reference
but the notion is “called for”

No explicit reference
hidden relevance of the notion

Notions of this Talk

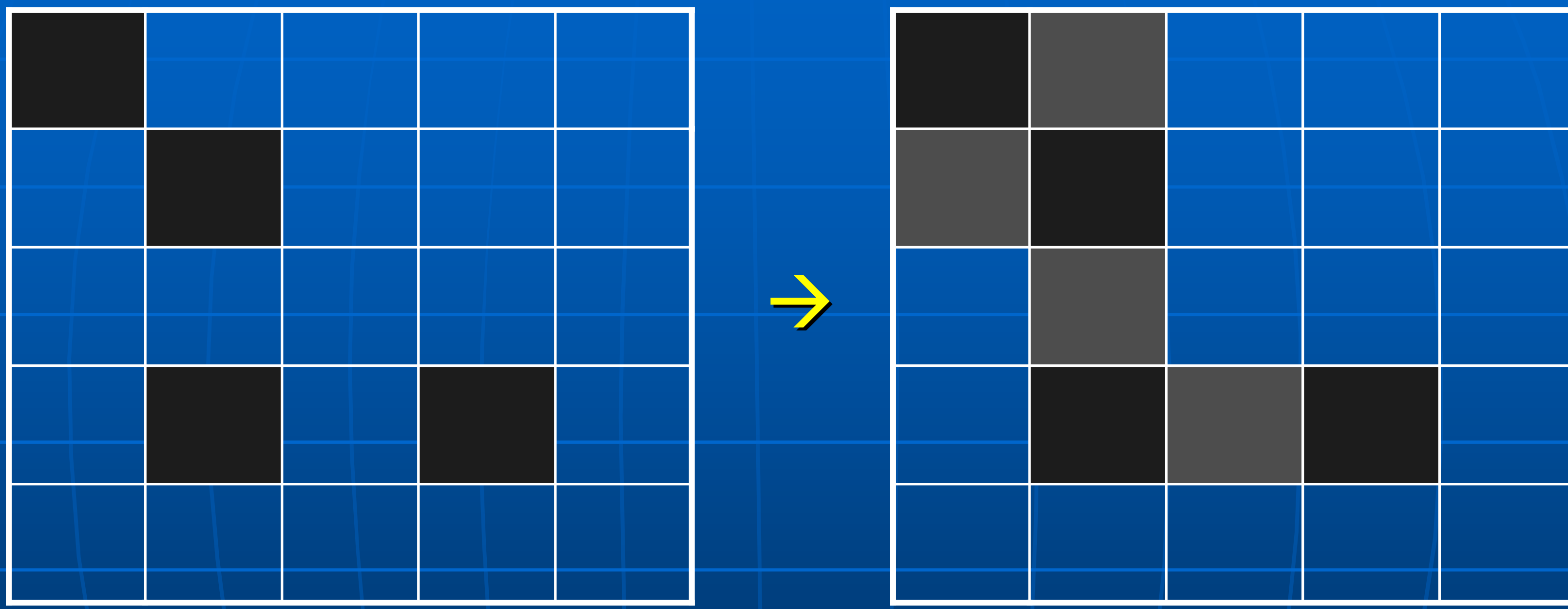
Rigor

in the design of argumentation

Induction ←→ Recursion

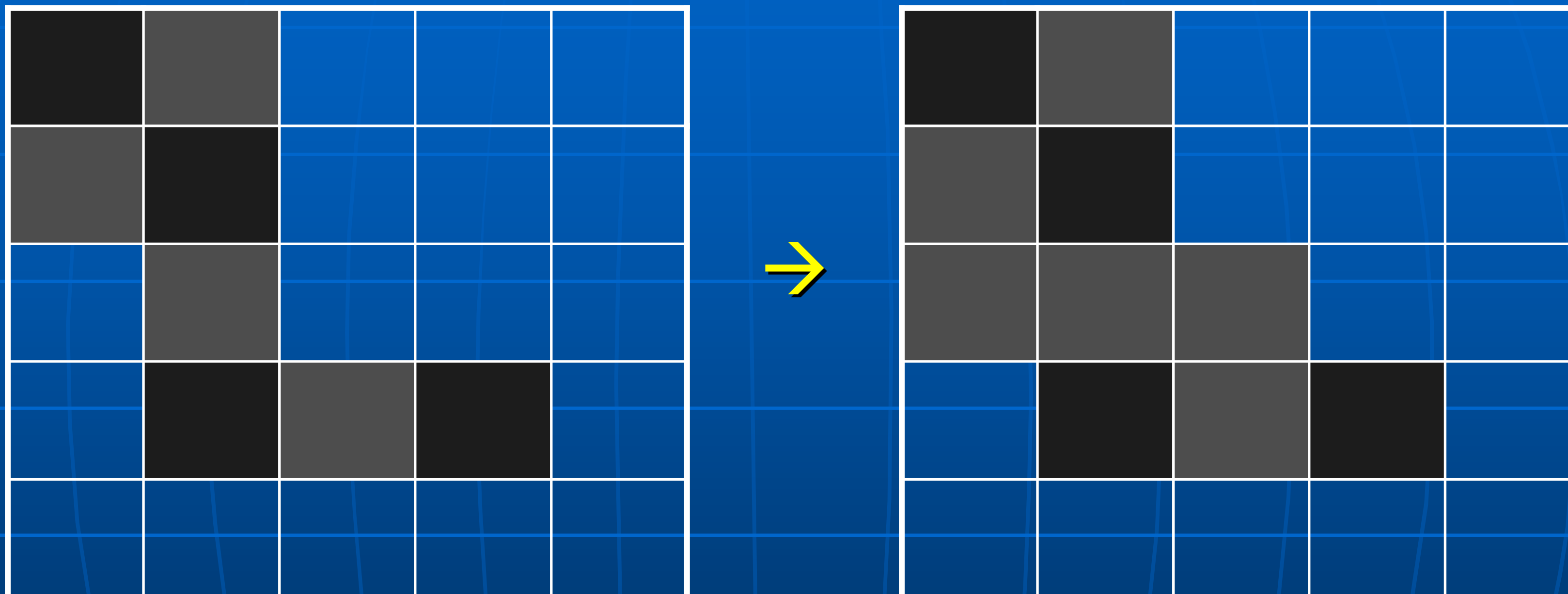
in the design of an algorithm

Board Staining



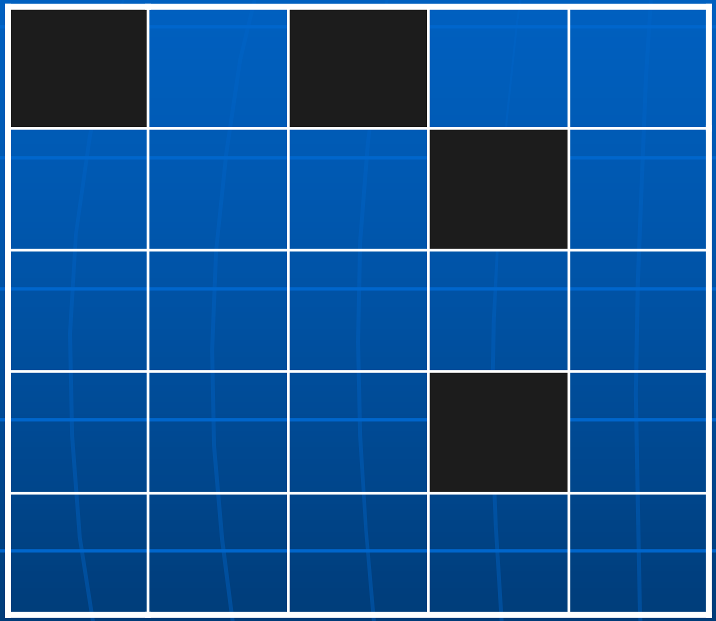
A board of N×N squares, N-1 are stained. A square with at least 2 stained neighbors becomes stained. Is there an initial staining that yields a stained board?

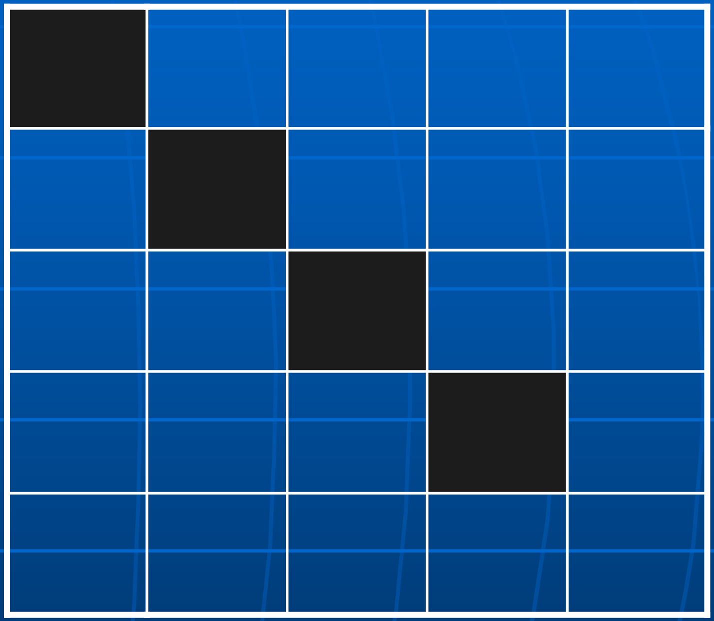
Board Staining



Eventually, only part of this board will be stained

Student (Teacher) Tendencies

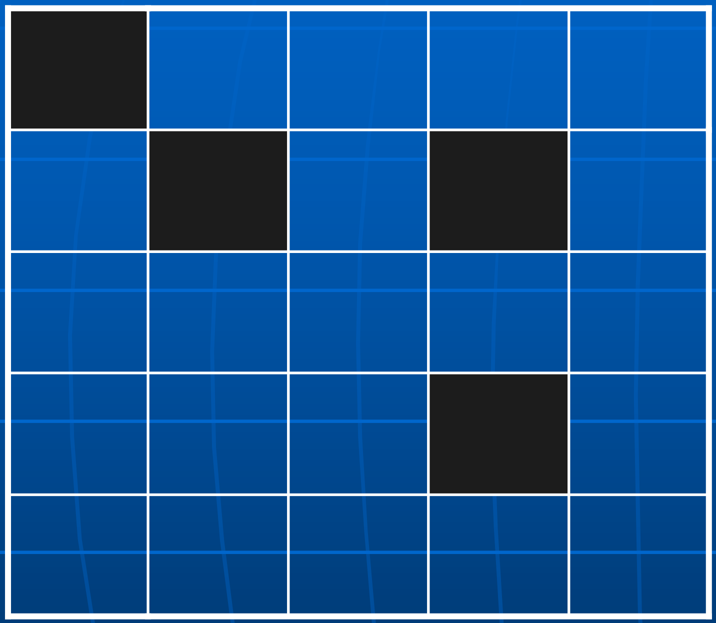


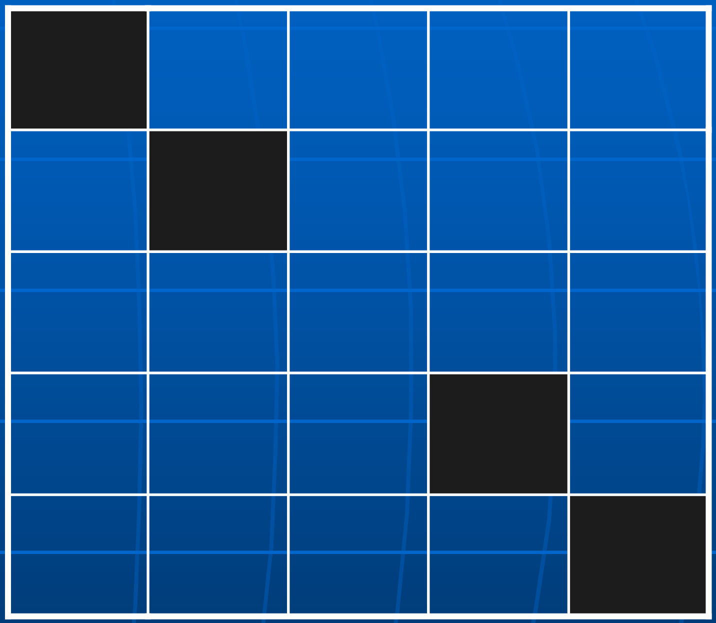


"Maximal initial structures, for which … no other structure may stain more …"

Seems true, but how do you prove that?

Student Tendencies





Try to prove by induction that “there will always be an unstained column and row”

Seems true, but how to apply the induction?

Student Tendencies

- Yield sound observations, but not patterns on which to capitalize

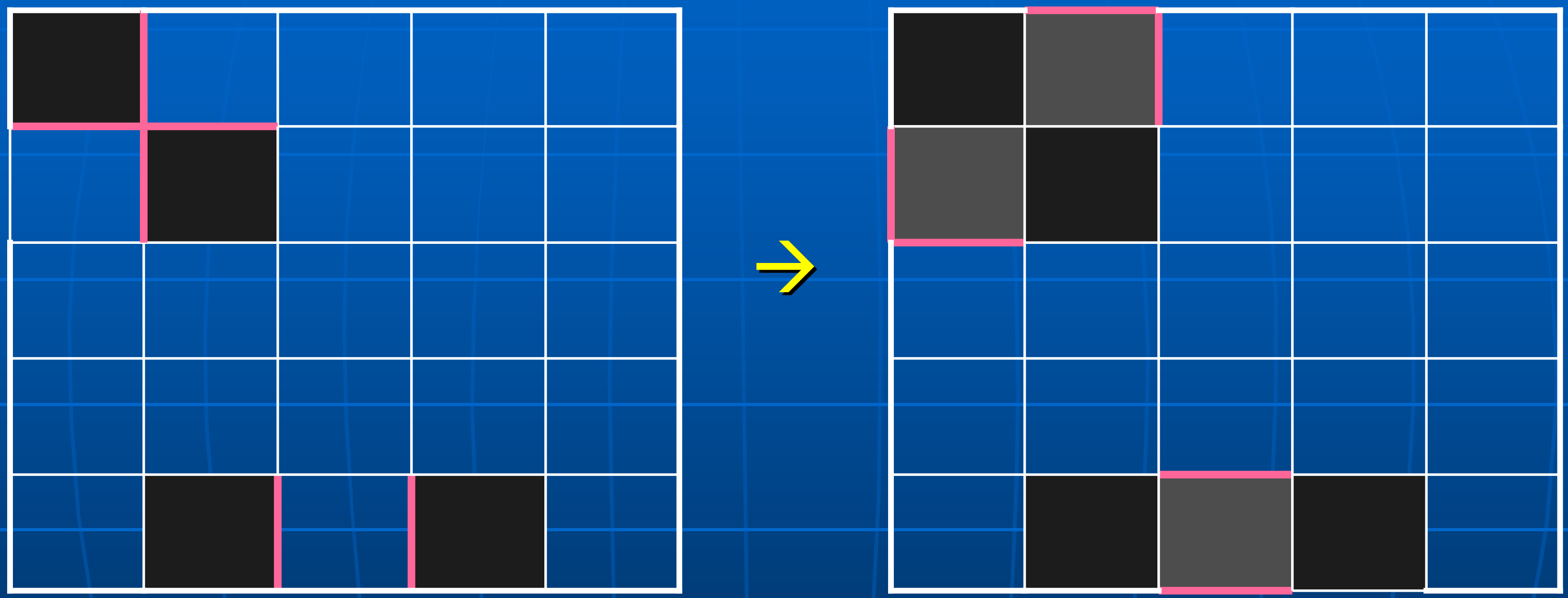
- Follow a single train of thought

- Do not view a proof construction as problem solving

→ Fixation, conflict → affective reaction

→ Cognitive tension between the clear observations and the inability to convince

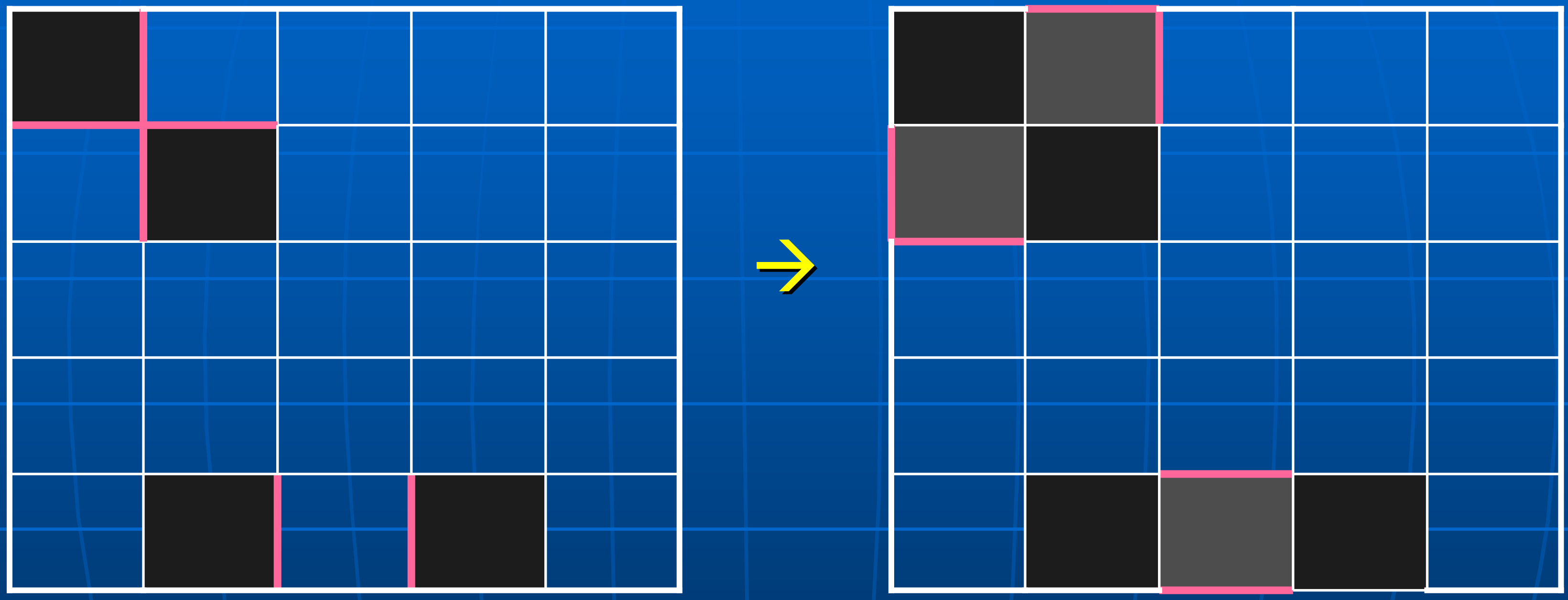
Change the Point of View



Sole area examination yields no clue

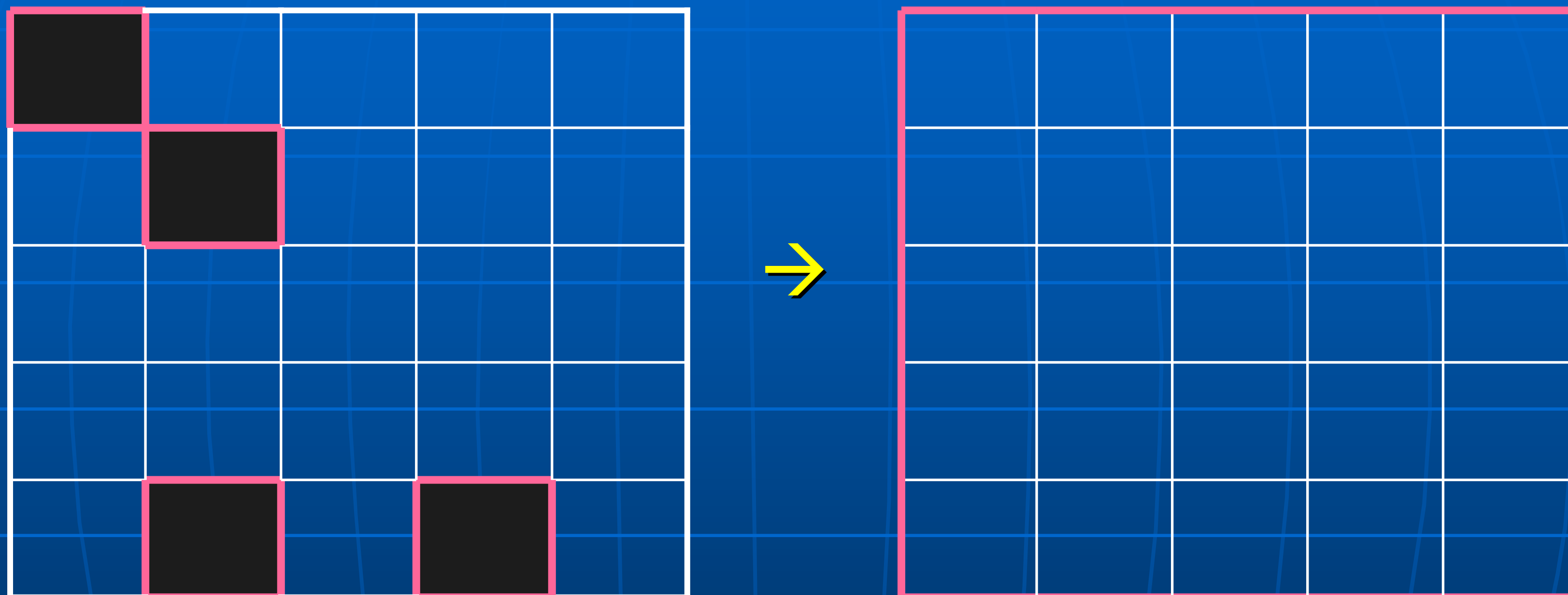
The circumference may also be relevant

Invariant Property



The number of stained circumference sides does not increase! → Invariant

Goal Cannot be Attained



Initially at most 4×(N-1) stained circum-sides

At the end they need to be 4×N. Impossible!

Learning

Role of rigor: a rigorous pattern yields convincing argumentation

Invariance property, and its link to the initial state and the final state

Relevance of attempting various points of view, not only the initial one

Learning by Conflict in Math

Infinity (e.g., Sierpinska, 1987)

{1, 2, 3, ... } ←→ {2, 4, 6, ...}

{1, 2, 3, ... } ←→ {(1,1), (1,2) ... (2,1) ...}

Epistemological obstacle (threshold concept?)

Proof elements (e.g., Movshovitz 1990)

Sqrt of 2 is irrational

→ Sqrt of 4 is irrational (deliberate errors)

Binary Sequence

w(1)=0   w(2)=001

w(i+1) is obtained from w(i) by replacing 0 by 001 and 1 by 0

→ w(3)=0010010

The value of the N-th bit in the first long-enough word?

Solution Attempts

The rules: 0→001, 1→0

w(1)=0, w(2)=001, w(3)=0010010

→ w(4)=0010010000010010001

Exponential growth

Solution approaches:

Inductive simulation, 1's locations(?)

Student Tendencies

Seek variants of inductive progression

... but the required space is too large

Seek patterns of the locations of 1's

... but no clear pattern

→ Fixation, Conflict

→ Cognitive tension, Epistemic curiosity

$w(4) = 0010010001001 0001$

Change the Point of View

The rules: 0→001, 1→0

w(1)=0, w(2)=001, w(3)=0010010

→ w(4)=0010010000100100001

→ w(i+1)=w(i)w(i)w(i-1)

Recursive view, inductive validation

Base: √ Step: w(i)=w(i-1)w(i-1)w(i-2)

Capitalize on the New Pattern

w(1)=0, w(2)=001, w(3)=0010010

w(4)=0010010001001000 1

w(i+1)=w(i)w(i)w(i-1)

→ length(i+1)=2×length(i)+length(i-1)

The length grows exponentially,
→ Keep a table of the word lengths

Compute Recursively

$w(1)=0$, $w(2)=001$, $w(3)=0010010$

$w(4)=0010010001001\,0001$

$L(2)=3$, $L(3)=7$, $L(4)=17$, $L(5)=41$

$w(i+1)=w(i)w(i)w(i-1)$

bit 20? $\rightarrow$ $17<20<41$ $\rightarrow$ $w(5)$

$w(5)=w(4)w(4)w(3)$ $\rightarrow$ bit 3 in $w(4)$

$w(4)=w(3)w(3)w(2)$ $\rightarrow$ bit 3 in $w(3)$

Learning

Induction ←→ Recursion

Opposite directions

But very close, incremental reasoning

Shown separately in CS studies

Induction in iteration and proofs

Recursion in reverse computations and data structures

Learning

But they may be relevant together:

Observing: $w(i+1)=w(i)w(i)w(i-1)$
by recursion (proving it by induction)

Constructing: $L(i+1)=2\times L(i)+L(i-1)$
by induction

Computing the N-th bit: by recursion
on the table of L's

Sign Switching

-3	7	9	-6	-8
-4	-6	-7	-8	9
9	-9	7	-5	7
-5	9	-8	3	6
-8	5	0	9	-7

→

3	-7	-9	6	8
-4	-6	-7	-8	9
9	-9	7	-5	7
-5	9	-8	3	6
-8	5	0	9	-7

Operator: may switch all signs in a row/column

Can you use the operator again and again and yield: all rows and columns sum to 0 or more?

Sign Switching

-3	7	9	-6	-8
-4	-6	-7	-8	9
9	-9	7	-5	7
-5	9	-8	3	6
-8	5	0	9	-7

→

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9	-9	7	-5	7
-5	9	-8	3	6
-8	5	0	9	-7

The top row was set, but
two columns were "damaged"

Student Tendencies

-3	7	9	-6	-8
-4	-6	-7	-8	9
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- Local point of view
- Seek explicit outcome at the “operated area”

Student Tendencies

Local viewpoint, no progress metric

→ Fixation, Conflict

Diverse attempts show that if one repeatedly applies the operator on a negative-sum line, eventually the goal is attained ... but, why?

→ Cognitive tension between the latter evidence and the inability to justify

Change the Point of View

-3	7	9	-6	-8
-4	-6	-7	-8	9
9	-9	7	-5	7
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- Seek a Global measure of progress

→ The sum of all the matrix numbers

Change the Point of View

-3	7	9	-6	-8
-4	-6	-7	-8	9
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- The sum of all the numbers increases
- It may not increase indefinitely

→ eventually successful termination

Learning

Seek a perspective

beyond the local one

Utilize a metric for progression

Realize “eventual” termination,

without a concrete scenario

of the progression steps

Conclusion

Recognize limited conceptual understanding of some notion

Select tasks that may yield impasse & conflict

Capitalize on the affective reaction and cognitive tension created

Utilize this tension to teach concepts, and possibly address epistemological obstacles